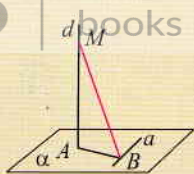


Teorema celor trei perpendiculare

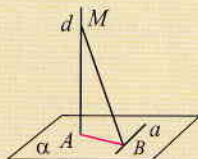
T₁₀ Teorema directă

$$\left. \begin{array}{l} d \perp \alpha \\ AB \perp a \\ AB, a \subset \alpha \end{array} \right\} \Rightarrow MB \perp a$$



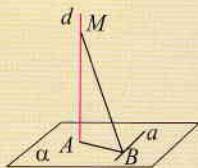
T₁₁ Teorema reciprocă 1

$$\left. \begin{array}{l} d \perp \alpha \\ MB \perp a \\ AB, a \subset \alpha \end{array} \right\} \Rightarrow AB \perp a$$



T₁₂ Teorema reciprocă 2

$$\left. \begin{array}{l} d \perp AB \\ AB \perp a \\ MB \perp a \\ AB, a \subset \alpha \end{array} \right\} \Rightarrow d \perp \alpha$$



Unghiul unei drepte cu un plan

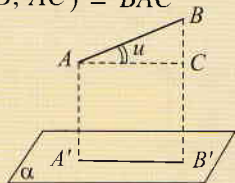
$\mathcal{P}_\alpha AB = A'B'$

$(AB, \alpha) = (\widehat{AB, A'B'}) = (\widehat{AB, AC}) = \widehat{BAC}$

unde $AC \parallel A'B'$

$m(\widehat{AB, \alpha}) = m(\widehat{BAC}) = u^\circ$

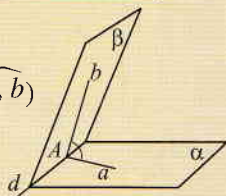
$A'B' = AB \cdot \cos u^\circ$



Unghi diedru

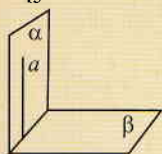
$$\left. \begin{array}{l} \alpha \cap \beta = d \\ a \perp d, a \subset \alpha \\ b \perp d, b \subset \beta \end{array} \right\} \Rightarrow (\widehat{\alpha, \beta}) = (\widehat{a, b})$$

dacă $m(\widehat{\alpha, \beta}) = 90^\circ \Rightarrow \alpha \perp \beta$



T₁₃ Plane perpendiculare

$$\left. \begin{array}{l} a \perp \beta \\ a \subset \alpha \end{array} \right\} \rightarrow \alpha \perp \beta$$

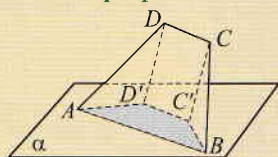


Proiecția unei figuri geometrice pe plan

$\mathcal{P}_\alpha(ABCD) = (A'B'C'D')$

$A_{\text{proiecției}} = A_{\text{figurii}} \cdot \cos u$

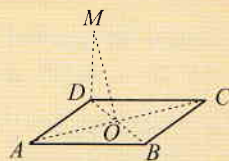
$u = m(\widehat{(ABCD), (A'B'C'D')})$



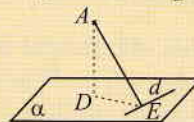
Calculul distanțelor

1. între două puncte = lungimea segmentului ce unește punctele

$M \notin (ABC), d(M, O) = MO$
 $AC \cap BD = \{O\}$

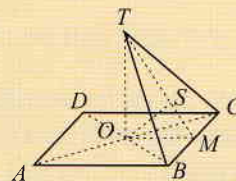


2. Distanța de la A la $d \subset \alpha$. se aplică T3 \perp



3. distanța de la un punct la un plan $d(O, (TBC))$.

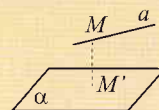
Se aplică R₂T3 \perp



4. distanța de la o dreaptă paralelă cu un plan, la plan, $a \parallel \alpha$

$M \in a$, ducem $MM' \perp \alpha$

$d(a, \alpha) = MM'$

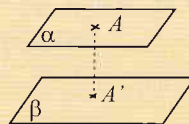


5. distanța între două plane paralele

$A \in \alpha$

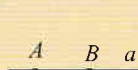
$\mathcal{P}_\alpha\{A\} = \{A'\}$

$d(\alpha, \beta) = AA'$



Unghiul a două drepte în spațiu

Drepte coplanare



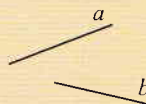
$m(\widehat{a, AB}) = 0^\circ;$

$a \cap b = \{M\}$

$m(\widehat{a, b}) = 0^\circ$

$m(\widehat{a, b}) = m\widehat{M}_1 \leq 90^\circ$

Drepte necoplanare



Fie S punct în spațiu

a, b drepte necoplanare

$\Rightarrow a' \parallel a; b' \parallel b,$
 $a' \cap b' = \{S\}$

atunci $m(\widehat{a, b}) = m(\widehat{a', b'}) \leq 90^\circ$.

„Nimic nu cade mai ușor sub simțuri decât figura (geometrică)”.

René Descartes

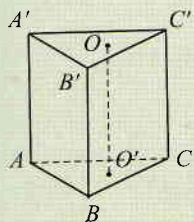
Prisma
 $A_l =$ suma ariilor fețelor laterale

$$A_l = A_l + 2A_b$$

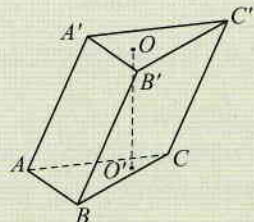
$$V = A_b \cdot h$$

Pentru prisma dreaptă $A_l = \mathcal{P}_b \cdot h$

\mathcal{P}_b perimetrul bazei;



Prismă dreaptă
 $h = AA'$



Prismă oblică
 $h < AA'$

• Prisme particulare

1. Paralelipiped dreptunghic (fețele sunt dreptunghiuri)

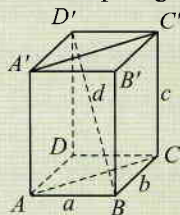
$$A_l = 2ac + 2bc$$

$$A_l = 2ab + 2ac + 2bc$$

$$V = a \cdot b \cdot c$$

$$d^2 = a^2 + b^2 + c^2$$

ACC'A' secțiune diagonală



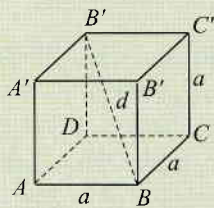
2. Cubul

$$A_l = 4a^2$$

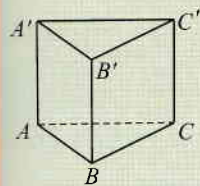
$$A_l = 6a^2$$

$$V = a^3$$

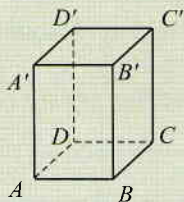
$$d = a\sqrt{3}$$



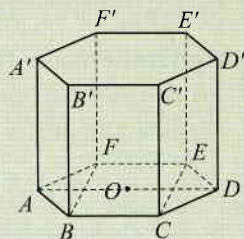
3. Prisma regulată (bazele sunt poligoane regulate)



triunghiulară



patrulateră



hexagonală

$$A_b = \frac{l^2 \sqrt{3}}{4}$$

$$A_b = l^2$$

$$A_b = \frac{3l^2 \sqrt{3}}{2}$$

4. Paralelipiped (fețele sunt paralelograme)

Piramida

$$A_l = A_l + A_b;$$

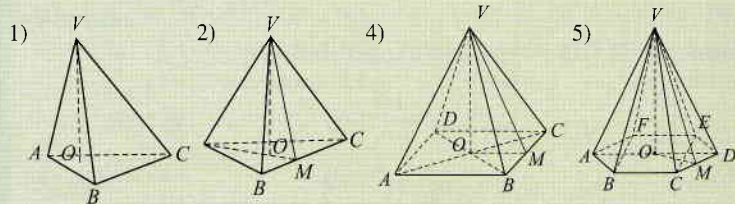
$$A_l = \frac{P_b \cdot a_p}{2};$$

$$V = \frac{A_b \cdot h}{3}$$

$A_l =$ suma ariilor fețelor laterale; a_p – apotema piramidei regulate

1) Piramida triunghiulară = tetraedru

2) Piramidă triunghiulară regulată ($\triangle ABC$ echilateral, $VA = VB = VC$)

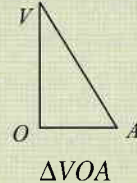
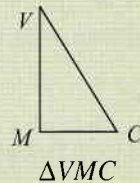
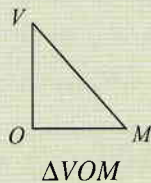


3) Tetraedru regulat ($VA = VB = VC = AB = AC = BC$)

4) Piramida patrulateră regulată (ABCD pătrat); $VM = a_p$.

5) Piramidă hexagonală regulată.

Triunghiuri dreptunghice importante la piramida regulată



$$(m(\widehat{VOM}) = 90^\circ) \quad (m(\widehat{VMC}) = 90^\circ) \quad (m(\widehat{VOA}) = 90^\circ)$$

VO – înălțimea; OM – apotema bazei

VM – apotema piramidei

OA – raza cercului circumscris

• Tetraedru regulat

$$VA = VB = VC = AB = BC = AC = l$$

$$h = \frac{l\sqrt{6}}{3}; \quad A_l = \frac{3l^2\sqrt{3}}{4};$$

$$A_l = l^2\sqrt{3}; \quad V = \frac{l^3\sqrt{2}}{12}$$

• Trunchiul de piramidă

$A_l =$ suma ariilor fețelor laterale

$$A_l = A_l + A_B + A_b$$

$$V = \frac{h}{3}(A_B + A_b + \sqrt{A_B \cdot A_b})$$

Pentru trunchiul de piramidă

$$\text{regulată } A_l = \frac{(\mathcal{P}_B + \mathcal{P}_b)a_{pt}}{2}$$

a_{pt} – apotema trunchiului de piramidă.

